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EQUIDISTANCE LINES AND THEIR USE IN STRUCTURAL CARTOGRAMS

Ekvidistanty a jejich použití ve strukturálních kartogramech. — Jako prostředek k znázornění, jak jsou v ohraničených areálech zastoupena statisticky zjištěná množství různých geografických hodnot nebo jejich jednotlivých druhově odlišných složek, lze užít i rozdělení areálu vnitřními ekvidistantami jeho hranic. Areál A lze rozdělit v různém poměru na vnitřní část C a vnější pás B (nebo na několik takových pásů) a na plochách B a C lze různými rozlišovacími prostředky vyjádřit další znaky. Pro československé okresy byla vyšetřena závislost ploch uzavřených vnitřními ekvidistantami okresních hranic na vzdálenosti ekvidistanty od hranice (isografické křivky) a vzdálenosti ekvidistant rozdělujících areál v poměru B : C byly tabulkovány (v km). Interpolací v tabulkách nebo na isografických křivkách můžeme určit kilometrovou hodnotu hledané ekvidistanty a zakreslit ji do kartogramu kteréhokoli měřítka. V připojeném kartogramu jsou uvnitř našich krajů zakresleny ekvidistanty, které vymezují tak velká území, aby obyvatelstvo kraje mělo v nich stejnou hustotu jako v kraji středočeském, kde je hustota zalidnění největší.

The term equidistance line is used to denote, in geography, lines of equal distances from the lines of a horizontal projection of the earth's surface.*) To be more precise, any geographically considered equidistance line is a curve any point of which has the property that its connecting lines with any point of the initial curve are equal to or longer than a given distance, with equality occurring in at least one case. This definition coincides, for smooth parts of the curve, with the definition of construction by means of normals and in edges it is by this definition that undesirable complications are eliminated. Equidistance lines are a special case of isarithms of geometrical continua.

The term *continuum* is used to denote phenomena which fill a surface or or a space in such a way that their state, value or intensity change continuously from point to point. Most of the continua are continua of spacial extent but cartographically primarily their characteristics on the earth's surface are represented. This is done by secants of surfaces of equal values of the continuum with a topographic surface. This is also the most common kind of isarithms in the field of physical-geographic sciences. However, even in these

*) In conformity with the mathematical terminology the term "equidistance" is used instead of "isodistance" as used in cartographic literature (see E. Imhof: Isolinienkarten; Intern. Jahrb. f. Kartogr., 1961).

sciences isarithms are dealt with which express the distribution in space of the continuum by contour lines of surfaces of equal values of it (e.g. by contour lines of isothermic surfaces in the atmosphere) and, of course, also isarithms of continua which are plainly continua of superficial extent.

Apart from concrete continua also *conventional* continua of both spacial and superficial extent exist which can also be represented by systems of isarithms. Such continua are produced, for instance, by any coordinate system chosen in a space or on a surface. Then definite coordinates belong to any point and for every coordinate there are surfaces or lines of equal values. Meridians and parallels are isarithms of geographical longitudes and latitudes, concentric circles in a plane isarithms of distance from a point in the plane, radii originating from that point isarithms of direction, etc. Also a distance from other lines of a projection of the earth's surface — fictitious as well as actual, natural as well as man-made — are a *geometrical* continuum and its isarithms, known as equidistance lines, have long been known and used in political, legal, economic and transportation contexts. Also belonging here are equidistance lines of coast lines, equidistance lines of towns (mileage right), equidistance lines of thoroughfares (isochoric curves) and many others.

Inside and outside of any closed curve a system of equidistance lines can be constructed. For the subject dealt with in this article we shall limit ourselves to *internal* equidistance lines. These can be looked upon as a system of isarithms from which a statistical surface can be formed and for the latter further statistical characteristics of the area enclosed by some curve can be derived, i. e. characteristics referred to that boundary.

A system of internal equidistance lines constructed at an arbitraly chosen but fixed mutual distance $\varrho = \Delta d$ reminds one of a system of isarithms of height in the neighbourhood of a negative peak of a topographic surface, i.e. a system of isobathes of an enclosed basi nwhich would have everywhere a uniform slope. This comparison is satisfied also by the fact that a statistical surface formed by equidistance lines of some enclosed curve splits up into parts intersecting along the edges produced by the shape of the zero equidistance line, i.e. of the boundary line itself. The majority of these edges (analogous to the valley lines of a negative shape of a topographic surface) terminates before reaching the negative peak of the surface given by the equidistance lines and only some of them actually meet in it.

Equidistance lines of any given line are constructed as *envelope lines* of circles of a radius $\varrho = n \,.\, \Delta d$, the centres of which are located on the initial line, or any subsequent internal equidistance line can be constructed from the preceding one by using $\varrho = \Delta d$, the latter being considered as a new initial line (an analogy of Huygens's explanation of wave-propagation in a homogeneous medium). The surfaces enclosed by internal equidistance lines constructed to the boundaries of administrative units (e.g. districts, regions, etc.) can be measured. It is further possible to construct graphs similar to hypsographic curves used for a representation of the dependence of the area of elevated positions of a topographic surface on the altitude above sea level. These so-called *isographic curves* representing the dependence of surfaces enclosed by equidistance lines on the magnitude of d of the equidistance lines permit the determination, for any area under examination, of the **mean distance** of its points from the boundary. For that purpose the same procedure is followed as for the determination of the mean altitude of a topographic surface from a hypsographic (or bathygraphic) curve.

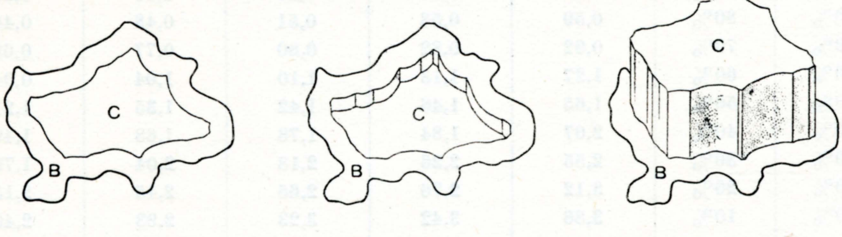

Fig. 1a,b,c.

Isographic curves have been constructed for all districts and regions in Czechoslovakia and this was done from the planimetric results on the maps of internal equidistance lines of districts with $\Delta d = 2$ km at a scale of 1 : 200 000 and on the maps of equidistance lines of regions with $\Delta d = 5$ km at a scale of 1 : 500 000. Neither the numerical values used for the construction of these graphs nor the graphs themselves are published but they have been used for the interpolation of the values listed in the tables which follow, for isographic curves can be used for the finding of equidistance lines which limit a definite part of the area A of the territory under examination.

Every internal equidistance line of the boundary line divides the figure into a boundary belt B and an inside part C. Therefore A = B + C. The ratio B : A will be designated by 1 : n. The distance d of the internal equidistnace line of a given figure separating the boundary belt $B = \frac{1}{n} A$ and enclosing the inside part $C = \frac{1}{n} A (n - 1)$ depends on the shape and size of the figure. In figures of an equal area every equidistance line separates a belt which is the larger, the more elongated and articulated the figure is. Thus, for instance, in a circle of an area of 100 cm² (r = 5.64 cm) a belt of an area B = C is separated by an equidistance line d = 1,65 cm², while a square of the same area is halved by an equidistance line d = 1,46 cm², a rectangle a = 2b by an equidistance line d = 1,35 cm, etc. For a square and a rectangle of an area A and a circumference p the distance d of an internal equidistance line limiting an inner part $C = \frac{1}{n} A (n - 1)$ can be calculated from the formula

$$d = \frac{1}{8}\left[p - \sqrt{p^2 - 16\,A\left(\frac{n-1}{n}\right)}\right];$$

demanding that K = x% A, we obtain

$$d = \frac{1}{8}\left(p - \sqrt{p^2 - 0.16 \times A}\right).$$

For A = 100 cm², d (in cm) is given in the table below.

B	C	Circle	Square	Rectangle		
				2a = 3b	a = 2b	a = 3b
10%	90%	0,29	0,26	0,25	0,23	0,22
20%	80%	0,59	0,53	0,51	0,48	0,46
30%	70%	0,92	0,82	0,80	0,77	0,69
40%	60%	1,27	1,13	1,10	1,04	0,94
50%	50%	1,65	1,46	1,42	1,35	1,21
60%	40%	2,07	1,84	1,78	1,68	1,49
70%	30%	2,55	2,26	2,18	2,04	1,79
80%	20%	3,12	2,76	2,65	2,45	2,15
90%	10%	3,86	3,42	3,23	2,93	2,46

For any area A′ = m . A the corresponding d″s are equal to $d\sqrt{m}$, e.g. for a rectangle, with a ratio of sides of 1 : 2 and A′ = 1600 cm² d′ of an equidistance line separating a 20% boundary belt is

$$d' = 0{,}48 \text{ cm} \times 4 = 1{,}92 \text{ cm}.$$

Territories which are required to be divided by equidistance lines into two or more parts at a definite proportion have, however, only exceptionally simple geometrical shapes such as were dealt with in the above formulas and table. At most it is sometimes possible to ascribe to them, to a rough approximation, one of these shapes and then to determine, also only to a rough approximation, the distance of the equidistance line sought. In most cases, however, it is necessary to proceed by the *method of isographic curves* which, for Czechoslovakia, yielded the following results.

Distance d (in km) for Czechoslovak Regions:

B = C =	10% 90%	20% 80%	30% 70%	40% 60%	50% 50%	60% 40%	70% 30%	80% 20%	90% 10%
Central Bohemian Region	2,05	4,40	6,88	9,70	12,70	16,25	20,50	25,58	31,95
Southern Bohemian Region	1,68	3,80	6,30	9,03	12,05	15,90	19,75	24,55	30,50
Western Bohemian Region	1,75	3,85	6,25	8,98	11,83	15,15	18,73	22,75	27,48
Northern Bohemian Region	1,08	1,45	4,13	5,73	7,50	9,53	12,03	14,70	17,98
Eastern Bohemian Region	1,58	3,38	5,33	7,53	10,05	12,93	16,25	20,58	25,90
Southern Moravian Region	2,03	4,55	7,45	10,50	13,70	17,20	21,25	25,80	31,75
Northern Moravian Region	1,50	3,48	5,78	8,50	11,38	14,55	18,20	22,05	26,00
Western Slovak Region	2,50	2,83	8,50	12,00	15,80	20,50	24,90	30,47	37,03
Central Slovak Region	2,38	4,88	7,80	11,25	15,05	19,15	23,93	29,53	37,70
Eastern Slovak Region	2,15	4,75	7,80	11,10	14,63	18,63	22,95	27,87	34,00

There are many graphical means to represent how statistically determined absolute quantities of various geographic values are distributed over the earth's surface — at definite points or in limited areas — and the relative representation of their individual distinctive kinds. As far as discontinuously distributed values are concerned (geographic discreta) structural *cartograms* and cartodiagrams are used in which the absolute quantities are represented by circles of radii proportional to the square roots of the quantities represented. By a division of the circles into sectors, the centre angle of which is proportional to the percentage of the qualitative components, several absolute data can be combined in one figure and their mutual proportion expressed. With choropleth cartograms we are limiting ourselves either to a single quantitative feature and graduate the same quantitatively (e.g. the percentage of agricultural land by a pattern or by a shade of colour) or we represent the proportions of several quantitative features of the phenomenon under consideration in the given area by division of this area into parallel stripes the width of which is proportional to the individual proportions (e.g. to the percentages of arable, forest and other land). Striped cartograms have the advantage that they facilitate the simultaneous perception of several features and that they replace several monothematic cartograms in which the concentrated colour of the individual stripes would be thinned by the pattern of the entire corresponding area of a choropleth cartogram.

The question arises whether, instead of a striped cartogram, which shows the proportional representation of the individual components accurately only by the width of the stripes (but not by their surface), it is not possible to use a *division of the area by internal equidistance lines of its boundaries.* In certain cases this is very easily possible and it presents, in fact, a very clear picture when a discontinuous phenomenon of surface is thus represented which is compared to the total surface of some territory (e.g. the above mentioned proportion of various kinds of utilization of the land). In such a case several equidistance lines will be used to separate several belts B and the remaining portion C. It depends on the nature of the situation to be represented and on the effect to be achieved by the cartogram to which component the outer belt will be allocated and to which one the inner part. Thus, for instance, when only forest land and its proportion to all land is being represented, it is of advantage to centralize the forest land in a compact area C while, when areas to be emphasized are areas bare of forests or deforested, waste lands etc., it is, on the other hand, more effective to allocate particularly to these negative, shortage components of the areas C empty, uncoloured windows of the cartogram; cf. fig. 16.*)

*) See Helburn-Edie Lightfoot: Montana in Maps, where this principle was applied for the differentiation between federal and country land.

Equidistance cartograms afford a possibility of marking, on surfaces B or C, the individual components in accordance with their proportional occurrence (e.g. the composition of forests by the stripe method).

Choropleth cartograms are used to represent, as already mentioned, mostly relative indicators (such as the percentage of the population belonging to industry). If two areas (districts) of such a cartogram receive the same shade of colour or the same pattern, it creates an erroneous impression of a quantity proportional to the area. Large areas of thinly populated mountain towns may receive high and even the highest shades of the scale expressing the proportion of the populations belonging to industry while small areas in densely populated territories receive the same or a lower value although high multiples of the industrial mountain population are concealed behind this value of the scale. Cartograms of occupations cannot be correctly interpreted without a simultaneous perception of a choropleth cartogram of the density of population.

Choropleth cartograms may mislead the reader in the same way as a beginner who lacks geographic experience is misled when he allows himself to be influenced only by the size of states and territories and attributes to them, in all respects, an importance in proportion to their size. This has long been realized by cartographers when they excluded, from certain special maps, uninhabited territories or attempted to produce *so-called anamorphated cartograms* in which each area was distorted and reduced in accordance with its importance. Thus, for instance, the sizes of territories were reduced in accordance with the population. The materialization of this highly logical concept is, of course, graphically difficult because the construction of the cartogram lacks geometric regularity.

The method of equidistance lines permits us to *replace an amorphated cartogram* without any disturbance of the concept of mutual position and shape of a territory. Every territorial unit of the cartogram can be reduced in surface by an equdistance line of the border. Even though the equidistance lines do not outline similar figures, the affinity of the original and of the derived figure is obvious. When a certain phenomenon is observed in a system of territorial units, the surfaces of the units can be reduced in accordance with the ratio of the value of each unit to the unit which is the maximum one of the whole system or to some other chosen comparative value. Thus we have reduced the regions of Czechoslovakia, each of which has a different density of population, by a comparison with the Central Bohemian Region which has the highest density, i.e. each region A was reduced by an equidistance line to $C = A \frac{h}{h_{max}}$

The population of each region was crowded together into an area enclosed by an equidistance line in such a way that the population then had, on this area, the density of the population of the Central Bohemian Region.

Only these surfaces can be filled with shades of colours or a pattern expressing some demographic or economic criteria linked with the population (e.g. the natural growth, the proportion of the population occupied in different fields of activities, etc.). In these areas the method of stripes can be applied, too.

However, even equidistance lines can be used to classify, in these artificially created areas with a uniformly concentrated population, the population by further criteria.

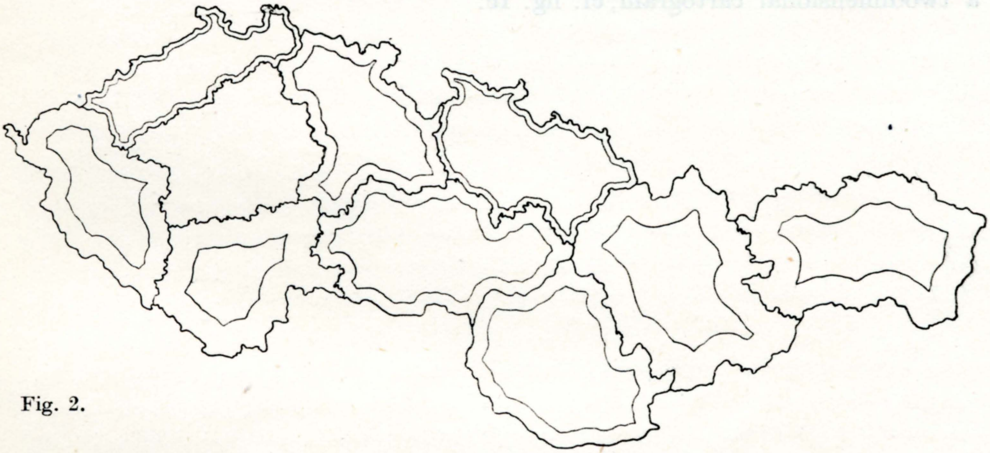

Fig. 2.

Very easily the question will arise, whether it is not possible to replace the division of surfaces by equidistance lines by division by geometric similarity, which is theoretically simple. However, arbitrariness occurs already in the choise of the centre of gravity, which does not affect the correctness of the reduction of the surface, but precludes uniqueness in the carrying out of the graphical work. In any case, however, a picture of unsatisfactory appearance is obtained in cases of intricate shapes of territories and the outlines of individual surfaces often touch or even overlap. Tests performed proved the unsuitableness of this method.

In certain applications the method of equidistance lines is limited by the belts B being too narrow when the central surface C is fairly large in comparison with the whole area. This happens when $B<30\%$ or $C>70\%$. This is also one of the factors which decide the sequence in which the parallel belts between equidistance lines should be used for the individual structural components when there are more of them. The outer belt has to be allocated to the component most strongly represented, the inner one to components of a lower relative representation. Equidistance structural cartograms are therefore not suitable for the representation of too small proportions, for they cannot be allocated, for the above reason, the belts B, but also not the central portion C, the boundary of which deviates, in this case, too much from the boundary of the area A.

The internal equidistance line (with a large d, i.e. for small proportions of C) sometimes split up into two parts and enclose two surfaces C_1 and C_2. In such a case the smaller one must be added to the larger one, i.e. d of the equidistance line of the surface C_1 must be increased and the area C_2 cancelled.

The surfaces C may become the bases for a three-dimensional cartogram (or built-up choropleth cartogram). This creates the possibility of using the height of columns for the expression of a further quantitative feature while there is a space, on their upper bases, for data which would be included in a twodimensional cartogram; cf. fig. 1c.